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INFLUENCE COEFFICIENTS FOR RADIATION IN A CIRCULAR CYLINDER*

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Abstract—A table of influence coefficients is calculated from which the distribution of absorbed radiation in a long circular cylinder having any distribution of emitted radiation can readily be determined. Lambert's law of diffuse emission and reflection is assumed, and the coefficient of absorbivity is supposed constant. A table of functions for the distribution of reflected radiation after each reflection up to the tenth is also given so that the results can be compared with non-Lambertian systems.

Résumé—Une table des coefficients d'influence a été calculée; elle permet de déterminer, par simple lecture, la distribution du rayonnement absorbé dans un long cylindre circulaire ayant une distribution de rayonnement émis quelconque. La loi de Lambert sur l'émission et la réflection diffuses est supposée valable et le coefficient d'absorption constant. Une table de fonctions pour la distribution du rayonnement réfléchi après chaque réflexion, jusqu'à la dixième, est également donnée, ce qui permet de comparer les résultats avec les systèmes non-Lambertien.

Zusammenfassung—Zur bestimmung der Verteilung der absorbierten Strahlung in einem langen Kreiszylinder mit beliebig verteilter emittierter Strahlung wird eine Tafel der Einflusskoeffizienten berechnet. Es wird das Lambertsche Gesetz für diffuse Emission und Reflexion zu Grunde gelegt und der Absorbtionskoeffizient konstant angenommen. Eine Tafel der Funktionen für die Verteilung der reflektierten Strahlung nach jeder Reflexion bis zur zehnten wird angegeben, so dass die Ergebnisse mit Systemen verglichen werden können, die nicht dem Lambertschen Gesetz gehorchen.

Аннотация—Приводится таблица коэффициентов взаимосвязи. При помощи данных, приведенных в таблице, можно определить распределение поглощенного излучения в длинном цилиндре кольцевого сечения с произвольным распределением испускаемого излучения. В основу исследования положен закон диффузной эмиссии Ламберта, при этом предполагается, что коэффициент поглащения является постоянным. Приводится также таблица функций для распределения отраженного излучения после каждого отражения вплоть до десятого. Результаты исследования автора можно сравнить с системами, не подчиняющимися закону Ламберта.

NOMENCLATURE

a,	coefficient of absorbivity;
c,	intensity of uniform radiation;
$f_0(\theta_0)$,	intensity of emitted radiation;
$f_n(\theta_n)(1-a)^n$,	intensity of radiation re-emitted after nth reflection;
$I(a, \theta)$,	intensity of absorbed radiation;
m,	number defining term in Fourier series;
n,	number of reflections;
r,	radius of cylinder;
α_{mn}, β_{mn},	coefficients in Fourier series;
θ,	angular co-ordinate;
ϕ,	angle between direction of radiation and normal to surface.

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INTRODUCTION

WHEN the walls of a closed container are not at a uniform temperature heat is transferred between them by a variety of means. There may be direct conduction of heat through the walls. If the enclosed space is filled with a gas or liquid there may be convective heat transfer. Finally, heat may be transferred from wall to wall by radiation. It is this last case that we propose to consider in the present report. If the temperatures are sufficiently high radiative heat transfer can become the most important mode.

The analysis is restricted to the case of a long circular cylinder, such as a missile, having a temperature distribution which varies arbitrarily with angular position but which is invariable along the length. Lambert's cosine law of diffuse radiation and reflection is assumed and the absorbivity is supposed constant. Five-figure tables of influence coefficients for the intensity of radiation absorbed at θ when a unit pulse is emitted at $\theta = 0$ are given for $\theta = 0(5)180°$ and absorbivity $= 0(0{\cdot}1)1$. Five-figure tables of the distribution of reflected radiation up to the tenth reflection are also given so that the results can readily be compared with those for non-Lambertian surfaces.

ANALYSIS

Consider a cylinder of radius r (Fig. 1). Let there be an initial emission of radiation from the inner surface of intensity $f_0(\theta_0)$. Then if we assume Lambert's cosine law of diffuse radiation an element $\delta\theta_0$ at A will radiate with an intensity $\frac{1}{2} f_0(\theta_0) r\delta\theta_0 \cos\phi$ per unit angle in a direction making an angle ϕ to the normal to the element

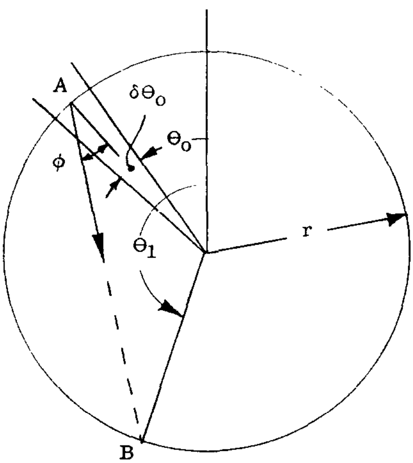

FIG. 1.

(Fig. 1). The intensity of radiation at B due to the emission at A will be $\frac{1}{2} f_0(\phi_0) r\delta\theta_0 \cos\phi / AB$, or, noting that AB makes an angle ϕ with the normal at B and putting $AB = 2r\cos\phi$, intensity of radiation normal to wall at B due to emission at A

$$= \tfrac{1}{4} f_0(\theta_0) \cos\phi\,\delta\theta_0. \tag{1}$$

The total intensity of radiation received at B due to the whole emission $f_0(\theta_0)$ can be obtained by integrating equation (1). It is important to notice that for $0 < \theta_0 < \theta_1$,

$$\cos\phi = \sin\left(\frac{\theta_1 - \theta_0}{2}\right)$$

and that for $\theta_1 < \theta_0 < 2\pi$,

$$\cos\phi = -\sin\left(\frac{\theta_1 - \theta_0}{2}\right).$$

If we denote the intensity of radiation received by $f_1(\theta_1)$, then

$$f_1(\theta_1) = \tfrac{1}{4}\int_0^{\theta_1} f_0(\theta_0)\sin\left(\frac{\theta_1 - \theta_0}{2}\right) d\theta_0 - \tfrac{1}{4}\int_{\theta_1}^{2\pi} f_0(\theta_0)\sin\left(\frac{\theta_1 - \theta_0}{2}\right) d\theta_0. \tag{2}$$

If we assume that the coefficient of absorbivity, a, is constant over the cylinder wall, then $af_1(\theta_1)$ will be absorbed and $(1 - a)f_1(\theta_1)$ will be re-emitted. If the re-emission also follows Lambert's law, then we can say that after n reflections there will be an emission

$$(1 - a)^n f_n(\theta_n) \tag{3}$$

where f_n is given by the general recurrence relationship

$$f_n(\theta_n) = \tfrac{1}{4}\int_0^{\theta_n} f_{n-1}(\theta_{n-1})\sin\left(\frac{\theta_n - \theta_{n-1}}{2}\right) d\theta_{n-1} - \tfrac{1}{4}\int_{\theta_n}^{2\pi} f_{n-1}(\theta_{n-1})\sin\left(\frac{\theta_n - \theta_{n-1}}{2}\right) d\theta_{n-1}. \tag{4}$$

By adding together the absorbed radiation at each reflection we find that the total intensity of radiation absorbed at θ due to the initial emission $f_0(\theta)$ is

$$I(a, \theta) = a \sum_{n=1}^{\infty} (1 - a)^{n-1} f_n. \qquad (5)$$

It is of interest to consider an emission after the nth reflection which can be expressed in terms of the Fourier series

$$(1 - a)^n f_n(\theta_n) = (1 - a)^n \,[c + \\ + \sum_{m=1}^{\infty} (\alpha_{mn} \sin m\theta_n + \beta_{mn} \cos m\theta_n)]. \qquad (6)$$

Applying equations (3) and (4) we find that the emission after the $n + 1$-th reflection is

$$\left.\begin{aligned} &(1 - a)^{n+1} f_{n+1}(\theta_{n+1}) = (1 - a)^{n+1} \Bigg[c \\ &- \sum_{m=1}^{\infty} \frac{1}{4m^2 - 1} (\alpha_{mn} \sin m\theta_{n+1} + \\ &\qquad\qquad + \beta_{mn} \cos m\theta_{n+1})\Bigg] \end{aligned}\right\}. \qquad (7)$$

Comparing equations (6) and (7) it will be seen that between the n-th and $n + 1$-th emissions there is an over-all reduction factor $(1 - a)$, and that apart from this the constant term c is unchanged and each of the harmonic terms is reflected with its sign changed and its magnitude diminished in the ratio $1/(4m^2 - 1)$.

The first harmonic is diminished at each reflection to $\frac{1}{3}$ of its previous value, the second to $\frac{1}{15}$, the third to $\frac{1}{35}$, and so on. It follows that whatever the form of the initial emission it quickly tends to uniformity and after a few reflections the emission can be represented as a constant term plus one or two terms of a Fourier series.

If the initial emission can be represented in the form

$$f_0(\theta_0) = c + \sum_{m=1}^{\infty} (\alpha_{m0} \sin m\theta_0 + \beta_{m0} \cos m\theta_0) \quad (8)$$

then by applying equations (5), (6) and (7) we find that the intensity of radiation absorbed at θ,

$$\left.\begin{aligned} &I(a,\theta) = \\ &c + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{\frac{(-1)^n a}{(1 - a)} \left(\frac{1 - a}{4m^2 - 1}\right)^n \right. \\ &\qquad \left. (\alpha_{m0} \sin m\theta + \beta_{m0} \cos m\theta)\right\} \\ &= c \sum_{m=1}^{\infty} \frac{a}{4m^2 - a} (\alpha_{m0} \sin m\theta + \\ &\qquad\qquad + \beta_{m0} \cos m\theta) \end{aligned}\right\}. \qquad (9)$$

By expressing a known distribution of emitted radiation, $f_0(\theta_0)$, in the form of a Fourier series, equation (9) can be used to determine the intensity of radiation received. The method is only useful, however, if the original Fourier series is fairly rapidly convergent since the additional factor $a/(4m^2 - a)$ does not hasten convergence greatly. For transient conditions, the necessity of expressing numbers of different distributions of radiation in series form would also be somewhat irksome. For these reasons it was decided to investigate another method of solution—the use of influence coefficients.

Suppose we have a line source of radiation at $\theta = 0$ of intensity r (this may alternatively be thought of as a source of unit strength in a cylinder of unit radius). Then the intensity of radiation received at θ will be the influence coefficient, $I(a, \theta)$. If we tabulate $I(a, \theta)$ the radiation received due to any initial distribution of emitted radiation can readily be determined, since the emitted radiation can be replaced by a few "lumped" outputs.

If we express our original emission at $\theta = 0$ in the form

$$f_0(\theta_0) = \frac{1}{2\pi} + \frac{1}{\pi} \sum_{m-1}^{\infty} \cos m\theta_0 \qquad (10)$$

then

$$I(a, \theta) = \frac{1}{2\pi} - \frac{1}{\pi} \sum_{m=1}^{\infty} \frac{a}{4m^2 - a} \cos m\theta. \quad (11)$$

The convergence of the series in equation (11) is unfortunately too slow for it to be of use for practical calculation.

Instead, it is easier to calculate the functions $f_1, f_2 \ldots$ from equation (4) and then to determine $I(a, \theta)$ from equation (5). Expressions for f_1–f_7 are given in Appendix I. It will be seen that the exact forms of these functions tend to become very cumbersome, but fortunately beyond f_6 or f_7 they are not needed. As we have previously noted, the higher harmonics of the radiation diminish rapidly with successive reflections and so by combining equation (10) with equation (4) we may write

Table 1. *Influence coefficient I* (a, θ)

$\theta°$	$a=0$	$a=0·1$	$a=0·2$	$a=0·3$	$a=0·4$	$a=0·5$	$a=0·6$	$a=0·7$	$a=0·8$	$a=0·9$	$a=1·0$
0	0·15915	0·14584	0·13207	0·11780	0·10299	0·08760	0·07158	0·05488	0·03743	0·01916	0
5	0·15915	0·14692	0·13423	0·12104	0·10731	0·09301	0·07808	0·06247	0·04612	0·02896	0·01090
10	0·15915	0·14797	0·13633	0·12421	0·11156	0·09834	0·08450	0·06999	0·05475	0·03871	0·02179
15	0·15915	0·14899	0·13839	0·12731	0·11571	0·10356	0·09081	0·07741	0·06329	0·04839	0·03263
20	0·15915	0·14998	0·14039	0·13033	0·11978	0·10870	0·09703	0·08473	0·07174	0·05799	0·04341
25	0·15915	0·15095	0·14233	0·13328	0·12376	0·11372	0·10313	0·09193	0·08007	0·06749	0·05411
30	0·15915	0·15188	0·14423	0·13616	0·12764	0·11864	0·10911	0·09901	0·08829	0·07687	0·06470
35	0·15915	0·15279	0·14607	0·13896	0·13143	0·12345	0·11498	0·10596	0·09637	0·08613	0·07518
40	0·15915	0·15367	0·14785	0·14167	0·13511	0·12814	0·12070	0·11277	0·10430	0·09523	0·08551
45	0·15915	0·15451	0·14957	0·14431	0·13870	0·13270	0·12629	0·11943	0·11208	0·10417	0·09567
50	0·15915	0·15533	0·15124	0·14687	0·14218	0·13714	0·13174	0·12593	0·11968	0·11294	0·10565
55	0·15915	0·15612	0·15285	0·14934	0·14554	0·14145	0·13704	0·13227	0·12710	0·12151	0·11544
60	0·15915	0·15688	0·15441	0·15172	0·14880	0·14563	0·14218	0·13842	0·13433	0·12987	0·12500
65	0·15915	0·15761	0·15590	0·15402	0·15195	0·14967	0·14715	0·14439	0·14135	0·13801	0·13432
70	0·15915	0·15831	0·15734	0·15623	0·15498	0·15356	0·15196	0·15017	0·14816	0·14591	0·14339
75	0·15915	0·15898	0·15871	0·15835	0·15789	0·15731	0·15660	0·15575	0·15474	0·15356	0·15219
80	0·15915	0·15962	0·16003	0·16039	0·16068	0·16091	0·16106	0·16112	0·16109	0·16095	0·16070
85	0·15915	0·16023	0·16128	0·16232	0·16335	0·16435	0·16533	0·16628	0·16719	0·16807	0·16890
90	0·15915	0·16080	0·16248	0·16417	0·16589	0·16764	0·16941	0·17121	0·17304	0·17489	0·17678
95	0·15915	0·16135	0·16361	0·16593	0·16831	0·17077	0·17330	0·17592	0·17862	0·18142	0·18432
100	0·15915	0·16186	0·16467	0·16758	0·17060	0·17374	0·17700	0·18039	0·18394	0·18764	0·19151
105	0·15915	0·16235	0·16568	0·16915	0·17276	0·17654	0·18049	0·18462	0·18897	0·19353	0·19834
110	0·15915	0·16281	0·16662	0·17061	0·17479	0·17917	0·18377	0·18861	0·19371	0·19909	0·20479
115	0·15915	0·16323	0·16750	0·17198	0·17669	0·18163	0·18685	0·19235	0·19816	0·20432	0·21085
120	0·15915	0·16362	0·16832	0·17325	0·17845	0·18392	0·18971	0·19582	0·20231	0·20919	0·21651
125	0·15915	0·16399	0·16907	0·17442	0·18007	0·18604	0·19235	0·19904	0·20614	0·21370	0·22175
130	0·15915	0·16432	0·16976	0·17549	0·18156	0·18798	0·19478	0·20199	0·20967	0·21785	0·22658
135	0·15915	0·16462	0·17038	0·17647	0·18291	0·18973	0·19698	0·20468	0·21288	0·22162	0·23097
140	0·15915	0·16488	0·17094	0·17734	0·18412	0·19131	0·19895	0·20709	0·21576	0·22502	0·23492
145	0·15915	0·16512	0·17143	0·17811	0·18519	0·19271	0·20070	0·20922	0·21831	0·22802	0·23843
150	0·15915	0·16533	0·17186	0·17877	0·18612	0·19392	0·20222	0·21108	0·22053	0·23064	0·24148
155	0·15915	0·16550	0·17222	0·17934	0·18690	0·19495	0·20351	0·21265	0·22241	0·23287	0·24407
160	0·15915	0·16564	0·17251	0·17980	0·18755	0·19579	0·20457	0·21394	0·22396	0·23469	0·24620
165	0·15915	0·16575	0·17274	0·18016	0·18805	0·19645	0·20539	0·21495	0·22517	0·23611	0·24786
170	0·15915	0·16583	0·17291	0·18042	0·18841	0·19692	0·20598	0·21567	0·22603	0·23713	0·24905
175	0·15915	0·16588	0·17301	0·18058	0·18863	0·19720	0·20634	0·21610	0·22654	0·23774	0·24976
180	0·15915	0·16589	0·17304	0·18063	0·18870	0·19729	0·20646	0·21624	0·22672	0·23794	0·25000

Table 2. Functions f

$\theta°$	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
0	0·00000	0·19635	0·14726	0·16309	0·15784	0·15959	0·15901	0·15920	0·15914	0·15916
5	0·01090	0·19617	0·14731	0·16308	0·15785	0·15959	0·15901	0·15920	0·15914	0·15916
10	0·02179	0·19563	0·14745	0·16303	0·15786	0·15958	0·15901	0·15920	0·15914	0·15916
15	0·03263	0·19476	0·14768	0·16296	0·15789	0·15958	0·15901	0·15920	0·15914	0·15916
20	0·04341	0·19359	0·14800	0·16285	0·15792	0·15957	0·15902	0·15920	0·15914	0·15916
25	0·05411	0·19213	0·14841	0·16272	0·15797	0·15955	0·15902	0·15920	0·15914	0·15916
30	0·06470	0·19040	0·14890	0·16256	0·15802	0·15953	0·15903	0·15920	0·15914	0·15916
35	0·07518	0·18844	0·14947	0·16238	0·15808	0·15951	0·15904	0·15919	0·15914	0·15916
40	0·08551	0·18626	0·15011	0·16217	0·15815	0·15949	0·15904	0·15919	0·15914	0·15916
45	0·09567	0·18389	0·15083	0·16193	0·15823	0·15946	0·15905	0·15919	0·15914	0·15916
50	0·10565	0·18135	0·15160	0·16168	0·15831	0·15944	0·15906	0·15919	0·15914	0·15916
55	0·11544	0·17867	0·15243	0·16141	0·15840	0·15941	0·15907	0·15918	0·15915	0·15916
60	0·12500	0·17586	0·15332	0·16112	0·15850	0·15937	0·15908	0·15918	0·15915	0·15916
65	0·13432	0·17296	0·15424	0·16081	0·15860	0·15934	0·15909	0·15918	0·15915	0·15916
70	0·14339	0·16999	0·15520	0·16049	0·15871	0·15930	0·15911	0·15917	0·15915	0·15916
75	0·15219	0·16696	0·15619	0·16017	0·15882	0·15927	0·15912	0·15917	0·15915	0·15916
80	0·16070	0·16391	0·15720	0·15983	0·15893	0·15923	0·15913	0·15916	0·15915	0·15916
85	0·16890	0·16085	0·15822	0·15949	0·15904	0·15919	0·15914	0·15916	0·15915	0·15916
90	0·17678	0·15781	0·15925	0·15915	0·15916	0·15915	0·15916	0·15915	0·15915	0·15915
95	0·18432	0·15480	0·16027	0·15881	0·15927	0·15912	0·15917	0·15915	0·15916	0·15915
100	0·19151	0·15185	0·16129	0·15847	0·15938	0·15908	0·15918	0·15915	0·15916	0·15915
105	0·19834	0·14897	0·16228	0·15813	0·15949	0·15904	0·15919	0·15914	0·15916	0·15915
110	0·20479	0·14619	0·16325	0·15781	0·15960	0·15901	0·15920	0·15914	0·15916	0·15915
115	0·21085	0·14352	0·16419	0·15749	0·15971	0·15897	0·15922	0·15913	0·15916	0·15915
120	0·21651	0·14098	0·16509	0·15719	0·15981	0·15894	0·15923	0·15913	0·15916	0·15915
125	0·22175	0·13858	0·16594	0·15690	0·15991	0·15890	0·15924	0·15913	0·15916	0·15915
130	0·22658	0·13634	0·16674	0·15663	0·16000	0·15887	0·15925	0·15912	0·15917	0·15915
135	0·23097	0·13427	0·16749	0·15638	0·16008	0·15885	0·15926	0·15912	0·15917	0·15915
140	0·23492	0·13239	0·16817	0·15615	0·16016	0·15882	0·15927	0·15912	0·15917	0·15915
145	0·23843	0·13070	0·16878	0·15594	0·16023	0·15880	0·15927	0·15912	0·15917	0·15915
150	0·24148	0·12921	0·16932	0·15575	0·16029	0·15878	0·15928	0·15911	0·15917	0·15915
155	0·24407	0·12794	0·16978	0·15560	0·16034	0·15876	0·15929	0·15911	0·15917	0·15915
160	0·24620	0·12689	0·17016	0·15547	0·16039	0·15874	0·15929	0·15911	0·15917	0·15915
165	0·24786	0·12607	0·17047	0·15536	0·16042	0·15873	0·15930	0·15911	0·15917	0·15915
170	0·24905	0·12548	0·17068	0·15529	0·16044	0·15873	0·15930	0·15911	0·15917	0·15915
175	0·24976	0·12512	0·17081	0·15525	0·16046	0·15872	0·15930	0·15911	0·15917	0·15915
180	0·25000	0·12500	0·17086	0·15523	0·16046	0·15872	0·15930	0·15911	0·15917	0·15915

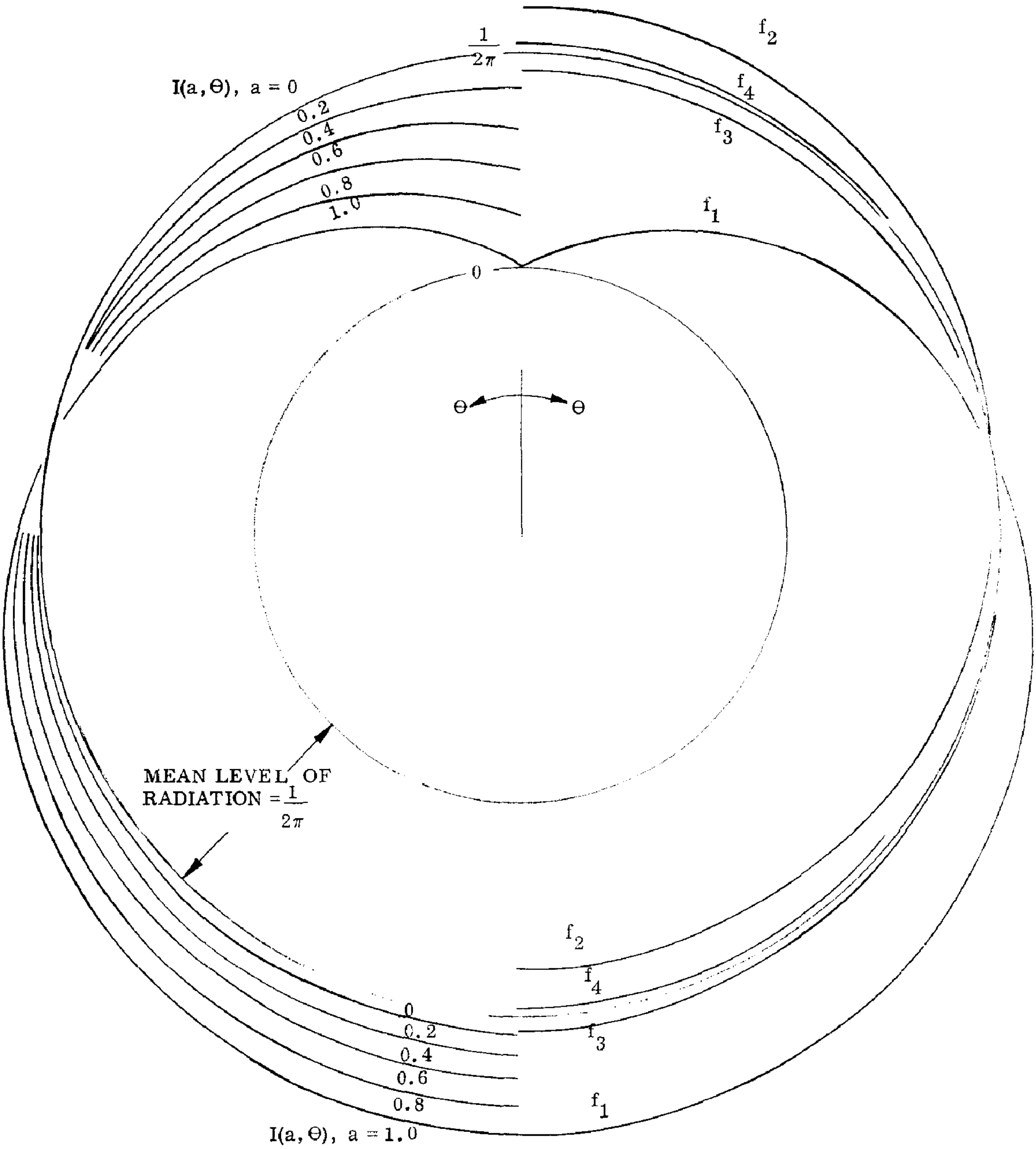

FIG. 2. Influence coefficients and functions *f*.

$$\left.\begin{aligned} f_7 &= \frac{1}{2\pi} - \frac{1}{3^7\pi}\cos\theta + --- \\ f_8 &= \frac{1}{2\pi} + \frac{1}{3^8\pi}\cos\theta + --- \\ f_9 &= \frac{1}{2\pi} - \frac{1}{3^9\pi}\cos\theta + --- \\ f_{10} &= \frac{1}{2\pi} + \frac{1}{3^{10}\pi}\cos\theta + --- \end{aligned}\right\} \quad (12)$$

where the terms neglected are less than 10^{-8}.

Our method of calculation is thus a hybrid. We use exact values of the functions f up to f_7 and complete the expression by using values of $f_8 --- f_\infty$ derived from Fourier analysis. The radiation absorbed up to the seventh reflection may be written $a \sum_{n=1}^{n=7} (1 - a)^{n-1} f_n$. If we consider the new emission after the seventh reflection (from equation (12)),

$$(1 - a)^7 \left\{\frac{1}{2\pi} - \frac{1}{3^7\pi}\cos\theta\right\}$$

then the contribution to $I(a, \theta)$ from it may be obtained from equations (10) and (11) as

$$(1 - a)^7 \frac{1}{2\pi} + \frac{a(1 - a)^7}{4 - a}\,\frac{1}{3^7\pi}\cos\theta.$$

In total, then, we have

$$I(a, \theta) = a \sum_{n=1}^{n=7} (1 - a)^{n-1} f_n + (1 - a)^7 \frac{1}{2\pi} + \\ + \frac{a(1 - a)^7}{4 - a}\,\frac{1}{3^7\pi}\cos\theta. \quad (13)$$

The functions f_1 to f_7 were calculated to seven decimal places using the expressions of Appendix I for $\theta = 0(5)180°$. $I(a, \theta)$ was then determined from equation (13), also to seven decimal places, for $a = 0(0{\cdot}1)1$. Finally the results were rounded down to give the five-figure table of $I(a, \theta)$ in Table 1. A separate five-figure table of f_1 to f_{10} (Table 2) is also given since the results may be useful for comparison with reflection from non-Lambertian surfaces: f_8, f_9 and f_{10} were calculated from equation (12).

RESULTS

The influence coefficient $I(a, \theta)$ for the intensity of radiation received at θ in a long cylinder of unit radius when a unit line emission occurs at $\theta = 0$ is tabulated to five figures in Table 1. Diffuse emission and reflection obeying Lambert's cosine law is assumed and the coefficient of absorbivity, a, is supposed constant. For high absorbivities $I(a, \theta)$ varies sinusoidally, for low absorbivities it tends to the constant value, $1/2\pi$.

The distribution of emitted radiation $f(\theta)$ after each of the first ten reflections is given to five figures in Table 2. The function $f(\theta)$ quickly tends to uniformity with successive reflections.

$I(a, \theta)$ and $f(\theta)$ are plotted in Fig. 2.

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APPENDIX I

The exact forms of the functions f_1 to f_7 are listed below:

	$\sin\frac{\theta}{2}$	$\theta(2\pi-\theta)\sin\frac{\theta}{2}$	$\theta^2(2\pi-\theta)^2\sin\frac{\theta}{2}$	$\theta^3(2\pi-\theta)^3\sin\frac{\theta}{2}$	$(\pi-\theta)\cos\frac{\theta}{2}$	$(\pi-\theta)^3\cos\frac{\theta}{2}$	$(\pi-\theta)^5\cos\frac{\theta}{2}$
f_1	$\frac{1}{4}$						
f_2	$\frac{1}{8}$				$\frac{1}{16}$		
f_3	$\frac{3}{32}$	$\frac{1}{128}$			$\frac{3}{64}$		
f_4	$\frac{5}{64}$	$\frac{1}{128}$			$\frac{5}{128}+\frac{\pi^2}{512}$	$-\frac{1}{1536}$	
f_5	$\frac{35}{512}$	$\frac{15}{2048}+\frac{\pi^2}{6144}$	$\frac{1}{24576}$		$\frac{35}{1024}+\frac{5\pi^2}{2048}$	$\frac{-5}{6144}$	
f_6	$\frac{63}{1024}$	$\frac{7}{1024}+\frac{\pi^2}{4096}$	$\frac{1}{16384}$		$\frac{63}{2048}+\frac{21\pi^2}{8192}+\frac{5\pi^4}{98304}$	$\frac{-7}{8192}-\frac{\pi^2}{49152}$	$\frac{1}{491{,}520}$
f_7	$\frac{231}{4096}$	$\frac{105}{16384}+\frac{7\pi^2}{24576}+\frac{\pi^4}{245{,}760}$	$\frac{7}{98304}+\frac{\pi^2}{983{,}040}$	$\frac{7}{11{,}796{,}480}$	$\frac{231}{8192}+\frac{21\pi^2}{8192}+\frac{35\pi^4}{393{,}216}$	$-\frac{7}{8192}-\frac{7\pi^2}{196{,}608}$	$\frac{7}{1{,}966{,}080}$